Spring 2018

How aspects of a modified Moore method in an upper-level, proof-intensive, collegiate mathematics course impact confidence among students

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How aspects of a modified Moore method in an upper-level, proof-intensive, collegiate mathematics course impact confidence among students

A Thesis Submitted to
the Faculty of the University of North Georgia
In Partial Fulfillment
Of the Requirements for the Degree
Bachelor of Science in Mathematics Education
With Honors

Katie Maynard

Spring 2018

Accepted by the Honors Faculty

of the University of North Georgia

in partial fulfillment of the requirements for the title of

Honors Program Graduate

Thesis Committee:

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Acknowledgements

Special thank you to Dr. Robb Sinn, Dr. Sherry Hix, Dr. Tanya Bennett, and Dr. Brad Bailey for their support and encouragement in the completion of this project.

Introduction

In pursuing a degree in the field of mathematics, there comes a point when a student is no longer solving problems, but rather constructing mathematical proofs. It is through these proofs that we are able to verify the validity of different mathematical concepts and formulas. One must understand and be able to prove his work if he hopes to make his mark on the field. Further, the kind of thinking developed in this process is necessary for successful performance in a number of fields, especially science, technology, engineering and math (STEM). However, how does one learn the art of proof writing? A proof cannot be memorized and reproduced like the quadratic formula. A proof requires deep thought and planning in order to make the reader understand the logistics behind the argument. This process cannot be taught like a traditional algebra or calculus class. The best method for thoroughly and effectively teaching these classes is where research and opinions begin to differ among educators and mathematicians.

In order to improve upon traditional lecture based mathematics courses, there have been numerous techniques developed and implemented which focus more on the students. These student centered methods have become known as inquiry-based learning (IBL). The first and most prominent IBL technique established in the field of mathematics was developed by Dr. Robert Lee Moore. Known as the Moore Method, his technique "prohibit[ed] students from using textbooks during the learning process, call[ed] for only the briefest of lectures in class and demand[ed] no collaboration or conferring between classmates" (Parker, 2005, vii). In this type of course focused on proof writing, students spend time outside of class constructing proofs and during each meeting present their results on the board. Since this strategy is a rather drastic change from what students are used to, few professors today implement the original Moore Method in its entirety. Instead, they use other inquiry-based techniques which are often referred to as Modified Moore Methods (MMMs).

An overview of the existing research on MMMs demonstrates trends focusing on how the method can be modified for graduate or undergraduate courses, how this teaching method affects students' views on mathematics, and how different modifications affect academic success. However, there are few studies analyzing how these methods impact students' confidence in their abilities within these unusually structured courses. Of the studies that have been conducted, the results show that students in IBL or MMM classes have less confidence than their peers in corresponding lecture-taught courses (Bailey et al., 2012, p. 395-396; Gormally et al., 2009, p. 14). This is especially concerning since studies have shown that mathematical confidence is strongly correlated to future career choice (Moakler & Kim, 2014).

If IBL and MMMs do increase students learning, then we need to make use of these strategies in order to help our students succeed. However, it is also important for students to remain confident in themselves within these courses. In order to see exactly how a modified Moore method impacts students' confidence, this study will break down the differences in MMM and lecture based instruction. By analyzing how these different aspects impact student confidence, I will determine what is most troublesome for students and suggest modifications that may increase understanding while promoting self-confidence.

Literature Review

Since Robert L. Moore first introduced his technique of teaching upper level mathematics courses in 1911, the Moore Method has been evaluated and modified by numerous mathematicians and professors. The modifications to the method first began with some of Moore's own students. These individuals took Moore's courses and then went on to become college professors themselves. The foundational study of the method was done by F.B. Jones (1977). Jones used the Moore method in his own classroom to teach courses in Topology as well as real and complex variable theory. Conventionally, these are the classes where the Moore method would be the most effective: courses requiring students to prove advanced theorems. Since those are the courses Jones taught, he was able to use the Moore method with little variation. In his classroom he catered to the students slightly more than the original method. For those students who were more shy and timid, Jones would write their proof on the board for them and allow them to present from their seats in the hopes that they could gradually gain the courage to present for themselves (Jones, 1977, p. 275). Jones also adjusted his students' required reading. After discovering that students had a hard time reading mathematics once they became accustomed to writing out solutions for themselves, Jones restricted the reading until after the middle of the term and then he refrained from discussing the reading in class (Jones, 1977, p. 276). Although these were only slight modifications, Jones made his class his own and adjusted the course to fit his students.

Not long after Jones published, other mathematicians also began to write about their own variations to the Moore Method. In 1982, David W. Cohen published his strategy for teaching a modified Moore method. Similar to Jones (1977), Cohen focused his technique on advanced courses like Hilbert spaces and "The Infinite" (Cohen, 1982, p. 488). In his courses, Cohen divided his classes into small groups of two or three and each week he assigned the groups a question (Cohen, 1982, p. 474). Unlike Jones (1977), Cohen's students only presented once a week. This gave him more time throughout the week to work with each group and assist them in their proof writing process (Cohen 1982, p. 474). This method greatly varies from that of R. L. Moore, however, Cohen (1982, p. 489) affirms that raising the communication level between the students, as well as with the teacher, helps to create a deeper understanding of the content. Cohen, of course, is not the only professor with this opinion. In his own study, Donald R. Chalice (1995, p. 319) expresses the need for small class sizes so that each student receives quality time presenting and getting feedback from their instructor and peers. However, Chalice's proposed method maintains more of Moore's original process than that of Cohen (1982). Chalice's (1995, p. 317-318)main modifications include: letting multiple students write on the board at a time, requiring definition exercises, creating three grading periods, and providing students with access to complete and correct versions of the proofs. Like Cohen, Chalice also focuses this method on his upper level mathematics courses, in this case intermediate analysis and advanced calculus.

In another study, W. T. Mahavier published his own take on the Moore method which was even further from the original. Mahavier's technique (1977) is similar to that of Cohen's (1982) in that the students only present once a week. However, unlike Cohen, Mahavier (1977, p. 134) uses his other two class periods for traditional lecture style instruction. What makes this strategy especially unique is that Mahavier is able to apply it to a larger range of courses. With less focus on proofs and presentations, this method can be applied to entry level courses such as college algebra, all the way up to calculus and numerical analysis (Mahavier, 1977, p. 134). Although, this method is far from Moore's original theory, it shows the modifications necessary to accommodate a larger variety of courses as well as a way in which the instructor is still responsible for providing a majority of the content.

None of the aforementioned researchers claim to have a perfect strategy for teaching any type of mathematics course. Although they attempt to get away from strict lecture based classes, they also recognize the difficulties that come from minimally guided instruction. One of the most commonly addressed difficulties is that of time restraints (Chalice, 1997, p. 318; Cohen, 1982, p. 474; Farnsworth, 2008, p. 693). When asking multiple students to present extensive and lengthy proofs on the board, the process of simply writing out their answers may take more than the allotted amount of time. Also, if a student does not correctly answer the question, the class must take more time to evaluate the presentation and correct the error. Such restraints can cause the class to get behind and not cover all the necessary content. Using this minimal guidance technique, there is also an issue when it comes to the students being the main teachers (Cohen, 1982, p. 474; Farnsworth, 2008, p. 695). Even if a student comes up with the correct response, they may not be able to accurately portray to their classmates the correct process and logic. This can be especially crucial if student presentations are the only way the content is covered for an upcoming test.

Another major challenge occurs when students come to class unprepared with nothing to present. In Cohen (1982) and Mahavier's (1997) modifications, this is not often a problem since students are only presenting once a week for a significant grade.

However, when using other techniques that more closely resemble the Moore method, students are presenting during each class period (Jones, 1977; Chalice, 1995). With a relatively large class, there would not be enough time for everyone to present on a given day. This may lead to students not actively participating and not coming to class with anything prepared. On some days it is possible that no one has anything to present and in order not to lose valuable class time the instructor must have something else arranged (Jones, 1977, p. 276). Additionally, it can be very difficult for students to present when their solutions have previously been rejected by the class (Farnsworth, 2008, p. 696; Jones, 1977, p. 276). Students may become embarrassed and lose confidence in themselves. It is important for the instructors to be mindful of these emotions and continue to encourage the students when they need extra help.

The final, major disadvantage is the strain the method puts on the instructor (Farnsworth, 2008, p. 695). Although the professor may not have to prepare a formal lesson for Moore method type classes, it is crucial that he is vigilant in critiquing the students' presentations. Since the student taught lessons are the main form of instruction, the teacher must make sure that they are completely accurate. Any misconceptions may lead to a misunderstanding that can hurt the students on tests and in future courses. However, it is also important for the instructors not to immediately intervene (Farnsworth, 2008, p. 695; Jones, 1977, p. 276). In order for the students to benefit from this minimal guidance method, they must put forth the effort to learn the material for themselves. If the instructor intervenes too early, the students may not gain the conceptual understanding that comes from self-discovery (Jones, 1977, p. 276). Also, if the students know that they will get help from the teacher whenever they have the slightest amount of trouble, they will no longer put forth the necessary amount of effort. It is a very difficult balance that the instructors must maintain in order to make the method beneficial to their students.

Although the previously reviewed articles are necessary for understanding the Moore method and the different modifications that have resulted, they do not provide statistical-based results to show they are effective methods of instruction. They are only helpful in stating the advantages and disadvantages of their respective processes. Cohen (1982, p. 489) states that, "… most students respond well to the responsibility placed on them by the Modified Moore method," and Farnsworth (2008, p. 696) claims that, "students appear to appreciate that they are learning some skills that they might not obtain otherwise." However, these assertions are not sufficient enough to justify the unconventional method of teaching. These researchers have not used experimental design or quantitative data to show that students respond well or succeed in this type of environment. For this reason, it is essential to look into other research in order to determine the probability of student success.

In order to gain a better understanding of the effects of the Moore method, Maya and Sumarmo (2011) conducted a study comparing two abstract algebra courses: one taught with a modified Moore method, the other in a traditional lecture format. The researchers analyzed how the different methods affected mathematical understanding, proving ability, and the students' perception of their respective courses (Maya & Sumarmo, 2011, p. 231). Maya and Sumarmo's (2011, p. 245) results showed that both the control and the modified Moore method group still struggled with constructing a well-written proof at the end of the semester and there was not a significant difference between the two groups in mathematical understanding. The main difference between the classes came from the opinion survey. Although both classes were satisfied with how the course was taught, students in the modified class were more comfortable participating, asking questions and working independently than the students in the traditional class (Maya & Sumarmo, 2011, p. 246). These results suggest that even though this modified Moore method was not necessarily helpful in terms of academic achievement, it did assist in developing a positive mathematical disposition that may help students in the future.

A similar study was conducted by Cooper, Bailey and Briggs in 2012. In their study, the researchers looked at three undergraduate Precalculus courses. They let one section be a modified Moore method class while the other two acted as control groups. Similar to the study conducted by Maya and Sumarmo (2011), Cooper, Bailey, and Briggs (2012, p. 390) gave their students the same test in order to compare their mathematical abilities as well as a survey to determine each group's opinion towards mathematics. This study's results varied from that of Maya and Sumarmo (2011), however, since Cooper et al. (2012, p. 395) found that students in the control group scored 10% lower on average than their peers in the modified Moore method class. Yet, students in the MMM course were more likely to underestimate their abilities on different tasks than those in the control group. Researchers even stated that, "the students in the control section drastically overestimated their abilities." Cooper et al. (2012, p. 395) also discovered that students in the MMM course reported to have less confidence going into Calculus than their peers in the control group. They also reported negative feedback from the students in the modified class.

> "They were terrified of making a mistake in front of all of their classmates.... Many students would come to class without solutions to the problem sequence because they truly believed that they were incapable of solving problems unless they had worked examples to mimic or their instructor showed them how to do so." (Cooper et al. 2012, p. 396).

The responses suggest that students were not fond of the modified Moore method class, but were more successful than the students taught in the lecture-based courses.

It is difficult to determine what accounts for these different results between the similar studies of Maya and Sumarmo and Cooper et al. One might consider that the classes were at different levels, Abstract Algebra being more advanced than Precalculus, or perhaps the difference in instructors' methods and attitudes should be analyzed. Regardless of the reasoning, more research is necessary to determine the effectiveness of minimally guided instruction on student learning.

In a separate article further analyzing part of the survey outcomes from their initial study, Bailey, Cooper, and Briggs (2012) dug deeper into the results to determine how a MMM affects "attitudes and beliefs about mathematics." Researchers found that between the MMM students and the control students, there was not a significant difference when comparing the survey items on their beliefs (Bailey et al, 2012, p.382). However, some statements revealed a significant difference of opinion between the genders. They also found that the opinions and attitudes from the students in the traditional style classes were more strongly correlated to their result on the final exam (Bailey et al., 2012, p. 382). This suggests that the students' attitude toward mathematics in the modified class did not reflect their mathematical capabilities whereas a low attitude in a lecture-based class implied a lower grade on the final.

Although these studies have their limits, they are creating a foundation for future studies to analyze how modified Moore methods impact students' academic success. However, they are not met without dispute. There are many researchers and educators who firmly believe that minimal guidance is not as effective as direct instruction. One of the most distinguished articles on this topic is by Kirschner, Sweller, and Clark (2006). Kirschner et al. (2006, p. 80) asserts that problem-solving based techniques, "…overburdens limited working memory and requires working memory resources to be used for activities that are unrelated to learning." This results in little knowledge accumulating in long-term memory since all the effort is working towards solving the problem. Kirschner et al. (2006, p. 80) suggests that the best way to form long-term memory is through repetition. Once students begin to recognize patterns, they can better recall the necessary techniques to solve a given problem. The article also emphasizes that minimal guidance techniques have not been proven to be effective for either upper or lower levels of instruction (Kirschner et al, 2006. p. 81). This idea significantly contradicts that of R. L. Moore and forces one to consider that the Moore method and its modifications may not always be the best strategy.

Although this view towards direct instruction is based on significant and thorough research, perhaps there are other ways to analyze direct instruction. In a 1989 study, Alan Schoenfeld surveyed 230 high school students to determine their views towards their math courses. The study revealed that the vast majority of students felt that mathematics is strictly objective and relies almost completely on memorization

(Schoenfeld, 1989, p. 344). Assuming these views are maintained as students transition into college, it is possible that direct instruction and memorization techniques are preferred simply because that is how students have always been taught. Requiring students who have always been lectured to develop concepts for themselves is a large and difficult step for any individual. In a study of remedial math courses, researchers discovered that few students had the necessary capability and understanding to succeed in a math class at the college level (Stage & Kloosterman, 1991, p. 33). Like those in the Schoenfeld (1989) study, these students also believed that memorization was essential for learning mathematics and they did not understand the importance of a conceptual based understanding (Stage & Kloosterman, 1991, p. 33). Stage and Kloosterman (1991, p. 30) found that there was a correlation between students' views of mathematics and their final grades. This relationship suggests that the students' focus on memorization and patterned processes inhibits their success in college mathematics courses. The researchers recommend that teachers in these remedial classes limit the amount of repetition and instead focus on conceptual development so that their students may be better prepared for future courses (Stage & Kloosterman, 1991, p. 34).

Increasing that focus on conceptual understanding will likely require changes to our traditional mathematics classrooms and any change to a students' idea of a normal classroom will likely cause some dispute between teacher and students. However, we have to implement these changes while also ensuring that students remain confident in their mathematical abilities. When students begin to doubt their capabilities, they often also second guess their choice of major and future career. One study analyzing factors that lead a student to select a STEM major found that mathematical confidence was a crucial component of that choice (Moakler & Kim, 2014, p. 139). Though researchers also investigated factors such as SAT scores, high school GPA, time spent studying, and having parents in STEM fields, none of these aspects were as instrumental in predicting a student's selected major (Moakler & Kim, 2014, p. 139). Moakler and Kim (2014, p. 139) state that "mathematics confidence clearly dominates the relationship with STEM major choice." These results suggest that even though a student may have strong mathematical capabilities, if he does not believe in those abilities he will likely not choose a STEM based major. Another study examining motivation in math as well as science found similar results. Aeschlimann, Herzog, and Makarova (2016) discovered that "improvement of the motivational conditions in mathematics, physics, and chemistry classes through targeted teaching practice not only can raise the learning motivation… but can also have a positive effect on their willingness to start studies in a STEM field." It is evident that motivation and confidence are essential for choosing a career in challenging STEM based fields and teaching methods play an instrumental role in those attitudes. For this reason it is necessary for teachers at all levels to work to encourage their students and promote mathematical confidence. How a course is taught greatly affects how the students respond to the content and how capable they feel with the material. Even strong mathematical students can be deterred from mathematics if they are not confident in their own abilities.

Since confidence in mathematics is so crucial to the pursuit of STEM careers, it is concerning that the study by Bailey et al. (2012) reveals a gap between student self-confidence and mathematical capabilities in an MMM structured course. Although it is important to note the apparent difference, it does not completely answer the question of why the MMM students felt less capable. Some of the low confidence revealed by the study may stem from the fact that researchers only looked at precalculus courses. Since precalculus is required for a variety of majors, not every student in a precalculus class will be especially strong in mathematics. However, what about students in upper-level mathematics courses? These students must be somewhat proficient in mathematics and confident in their abilities in order to pursue their desired major, whether that be mathematics itself or another STEM based field. One might assume that such students would not begin to doubt themselves simply because of a change in teaching style. However, there are other important factors that must be taken into consideration. In a MMM course, students are doing more work and often getting less feedback. Of the feedback that is provided, most is given by, and in front of their peers after their presentations. Although the instructor may feel that an individual is doing well in the course, the student is likely not as sure of himself. Yet, MMM has been shown to increase student understanding and achievement (Cooper et al., 2012, p. 395). In order to bridge this gap in confidence and achievement, my study analyzes a modified Moore method course to determine what exactly causes a lack of confidence.

Research Question

To expand on the previous findings of researchers in regards to the affects of a modified Moore method on students' mathematical beliefs and academic success, the following study aims to investigate the following questions:

What aspects of a modified Moore method prevent students' confidence within the MMM course?

How does previous experience in a MMM course impact students' confidence?

How do the two genders compare in regards to confidence within their MMM course?

Method

Due to limited offerings of modified Moore method courses at my University, a single upper-level geometry course was analyzed.

Course Structure

The Geometry class was a 3000 level, proof–intensive course focused on proving geometric theorems. The prerequisite for this course was Introduction to Mathematical Proofs so each student should have come into the class knowing the basics of proof writing. Class was held twice a week for an hour and fifteen minutes each session.

Grading. Students grades came from two tests worth 20% of the course grade, a final exam worth 30%, a proof portfolio worth 10%, and what were called "Moore Method proof points" that were worth 20%. The Moore Method proof points came from students daily proof presentations. The following criteria was given in the course syllabus:

Each proof will earn points as follows:

- 1 point – proof attempted but substantially incorrect.
- 2 points – proof attempted with several minor flaws or one major flaw, but overall proof outline is correct.
- 3 points – proof attempted and is substantially correct with only 1 or 2 minor flaws.

Up to three bonus points will be added to the proof points as follows:

- Proof is in LaTeX: +1.
- Sketch is Geogebra: +1.
- Proof is in a dynamic Geogebra file with LaTeX equation/text boxes where the sketch can be manipulated, and the proof steps appear one by one from beginning of the proof to the end: +1.

The proof portfolio was a project that students worked on throughout the semester. It required students to submit ten well-written proofs on the last day of class. The proofs had to meet certain requirements to show that students understood how to construct proofs over different topics and techniques. However, students were also assigned biweekly proofs for homework that were then assessed and returned to the student. This allowed the students to get feedback on their proofs before they submitted the portfolio at the end of the semester.

Daily routine. Each day the students would come to class with, ideally, two or three proofs ready to present. Three to four students would then go up to the board and begin writing out their proofs, or if they had it typed they could display it on the projector. Once the students completed their proof, each would take their turn presenting. After each presentation, the professor would award between one and three points based on the criteria given above. Over the course of the semester the students accumulated points which then corresponded to their Moore method proof points grade.

To encourage every student to present, initials of the three or four students with the least number of proof points were put on the board at the beginning of each class signifying that those students had priority that day. This gave more timid students an opportunity to present without feeling that they had to compete for a spot on the board.

In addition to this grade, the students also received a classwork grade for each day of class. The classwork grade came from the students' ability to critique one another's proof. Each student must be paying attention to the presentations in order to recognize any inconsistencies in a proof. After each presentation the class would be given a brief amount of time to evaluate the proof and if they were satisfied, the proof went to the instructor for approval. If the instructor found something wrong that the students did not catch, the class would lose five points from their classwork grade. Everyone in the class received the same classwork grade at the end of each day so they had to be vigilant as a group in order to keep up their score.

The instructor gave a few lessons as deemed necessary, however, the daily routine did not often differ from that previously outlined.

The Survey

In order to assess students' confidence within the course and what aspects impact those feelings, a survey was given after students took their midterm and received their scores back (see Appendix A). At this point in the semester, students should have had a good handle on the requirements of the course as well as an idea of how well they were doing in the class. They would have already formulated an opinion of this modified Moore method and their overall confidence within the course.

The survey consisted of 52 Likert type statements in which the students would responded on a scale of one to five how much they agreed or disagreed with each statement. A response of one was a strong disagreement, three was neutral, and five signified that students strongly agreed with the given statement. The survey was composed of sets of questions to assess the following:

- Perception of mathematics (statements from Schoenfeld, 1989)
- Confidence in one's mathematical abilities
- How students feel about six aspects of the course that most differ from lecture style courses:
    - The course structure
    - Homework requirements
    - Writing mathematical proofs
    - Daily presentations
    - The geometry content
    - The instructor
- Their overall opinion and confidence within the MMM course

The students' opinions of the instructor were analyzed to see that the individual professor did not have an effect on student confidence.

Once the surveys were completed, responses were compiled in an Excel spreadsheet. The data was analyzed specifically by looking at how those six aspects impacted students' confidence within the course.

Students were also asked to give some biographical information and an account of their previous experience with modified Moore method courses. This allowed me, the researcher, to analyze results based on gender as well as previous MMM experience. The last question of the survey asked whether or not they would be willing to participate in an interview in order to discuss their MMM course. The interviews were requested so that I could have a more thorough understanding of the students' views of the course and what aspects they believe should be changed in order to increase their self-confidence.

Class Demographics

Sixteen students were enrolled in this course, however, on the day the survey was administered, only fifteen students were in attendance and thus one student is not accounted for. Figure 1 outlines some aspects of the students within the class as assessed by the survey.

Figure 1:

Characteristics	Number of students
Gender	
Male	5
Female	9
No response	1
Age	
19-20	3
21-22	9
23 or older	3
Class	
Junior	6
Senior	9
Major	
Mathematics	8
Mathematics Education	6
Other (General Studies)	1
Previous MMM Experience?	
Yes	6
No	9

Results

In general, students seemed fairly confident in mathematics as well as their geometry course. The mean response to statement 11 (S11), "I feel confident in my mathematical abilities," was 3.9 (See Appendix B). Only one student disagreed with this statement, two gave a neutral response, and the remaining 12 students agreed. In a class mostly composed of mathematics and mathematics education majors, this mostly positive opinion was to be expected. In response to S48, "Overall, I feel that I am succeeding in my Geometry course" students had a mean response of 3.5. This statement was equated to confidence within the course. If students felt like they were succeeding then it is understood that they are confident. Although students were somewhat confident in the course, it is clear that there is room for improvement and increased self-assurance.

It is also interesting to note that in response to S47, "…I have a positive opinion of my MMM class," the average response was a 2.77. This suggests that students, in general, were not enjoying the course although they were more likely to report that they were confident within the course.

The means and standard deviations for each statement can be viewed in Appendix B along with the means when looking at gender, previous MMM experience.

Impacts on Confidence

A 52 × 52 correlation matrix (available upon request) was created in order to see which statements' responses were interrelated. However, since this studies focus is on the students' confidence, the correlation matrix was analyzed specifically looking for statements that had a moderate or strong correlation to statement 48, "Overall, I feel that I am succeeding in my geometry course," and 49, "Going into the midterm I felt confident in my proof writing abilities." The statements that had a correlation greater than or equal to |0.5| are displayed in Table 2.

Figure 2:

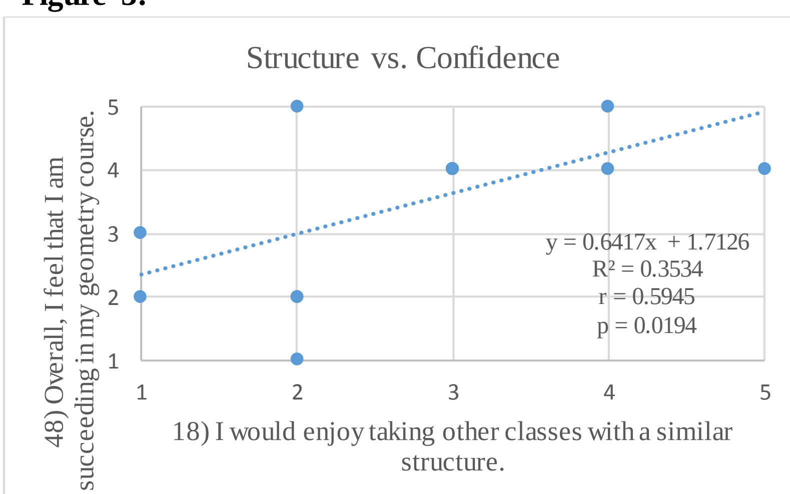
	Statement	**Correlation to S48**	**Correlation to S49**
S5	There is only one way to correctly answer a math problem	-0.52	-0.56
S9	Conceptual understanding is important for success in math.	-0.52	-0.57
S13	I feel comfortable in my MMM course.	0.64	0.75
S14	I like how the course is structured.	0.55	0.57

S15	The course is structure in the best way for me to learn the content.	0.60	0.62
S16	I learn more with a MMM structure than in a lecture based course.	0.59	0.51
S17	I prefer a lecture-based course.	-0.51	-0.38
S18	I would enjoy taking other math classes with a similar structure.	0.72	0.68
S22	My geometry homework is easier than that of most of my other upper-level math courses	0.54	0.57
S26	I do not do as much homework as I should for this class.	-0.60	-0.57
S28	My MMM instructor plays an important role in helping me learn.	0.66	0.72
S30	My instructor is very knowledgeable about the course content.	0.63	0.60
S31	I enjoy writing proofs.	0.77	0.85
S35	I enjoy presenting in my MMM class.	0.66	0.68
S37	Presentations are easy.	0.47	0.51
S38	I prefer student presentations over lectures from the instructor.	0.68	0.60
S39	My classmates are encouraging during the daily presentations in my MMM class.	0.39	0.55
S44	My MMM course encourages self-discovery.	0.68	0.59
S45	I prefer learning in a MMM environment.	0.61	0.58
S46	Compared to my lecture based math courses, I feel like my MMM class covers more content.	0.52	0.36
S47	Overall, I have a positive opinion of my MMM class.	0.66	0.63
S48	Overall, I feel that I am succeeding in my Geometry course.	1.00	0.89

It is not surprising to see that students comfort within the class (S13) is correlated to student confidence. However, to find the source of this discomfort, the researcher then looked at statements in relation to the six aspects presumed to affect students' confidence: the course structure, homework requirements, writing mathematical proofs, daily presentations, the geometry content, and the instructor.

Structure. Questions 14 through 18 were targeted to assess students' opinion of the structure of the course. Overall students seemed to dislike the structure of the course. A majority of the class, 11 out of 15, responded that they prefer a lecture-based course as compared to a MMM course (S17). Further, when asked if "the course [was] structured in the best way for [them] to learn the content," (S15) the mean response was a 2.5. Statement 16, "I learn more with a MMM structure than in a lecture based course" also demonstrated students dislike for the structure with a mean of 2.3. Of these four statements, statement 18 ("I would enjoy taking other math classes with similar structure") had the strongest correlation to students' confidence in the course (S48). Analyzing S18 in comparison to S48 using linear regression results in a moderate, positive correlation ($r = 0.59$). That implies that 35% of the variance of confidence in the course is accounted for by the students' views on the course structure ($R^2 = 0.35$). The moderate correlation makes it necessary for further analysis and so a linear regression t-test was performed at the 5% significance level ($\alpha = 0.05$). For this test, the null hypothesis, H_0, would be that the correlation parameter, ρ, would equal zero and the alternative hypothesis H_a, would be that $\rho \neq 0$. The test was run using a TI-84 and gave $p = 0.02$. Since $p < \alpha$, the null was rejected. Thus, there is sufficient evidence to suggest a significant linear relationship between students view on the course structure and their confidence within the course.

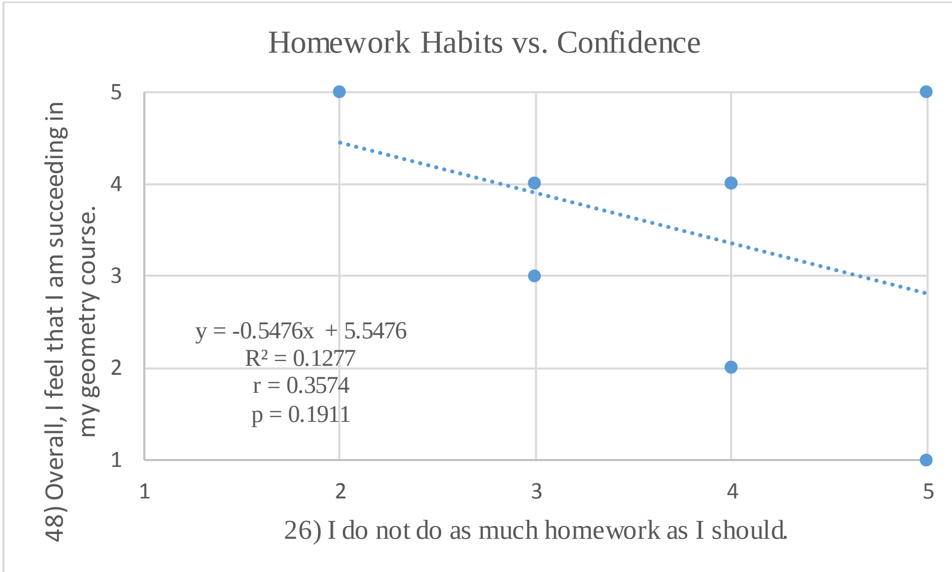
Figure 3:



Homework Requirements. To see how students feel about different and often more time consuming homework assignments, statements 20 through 26 assess these opinions. In response to S21, "compared to my lecture based course, I spend more time outside of class working on my MMM homework," students gave an average response of a 3.4. This suggests that students, on average, spend nearly the same amount of time on course work for this class as their others. Yet, there were no students who disagreed that they have to put more time into their MMM Geometry course in order to learn (S20: $\bar{x}$ = 3.9). Further, 11 out of 15 agreed that they do not do as much homework as they should (S20: $\bar{x}$ = 3.9). Although this response is likely not surprising to any mathematics teacher, it is interesting to note that students know they need to put forth more effort, but still neglect that increased responsibility.

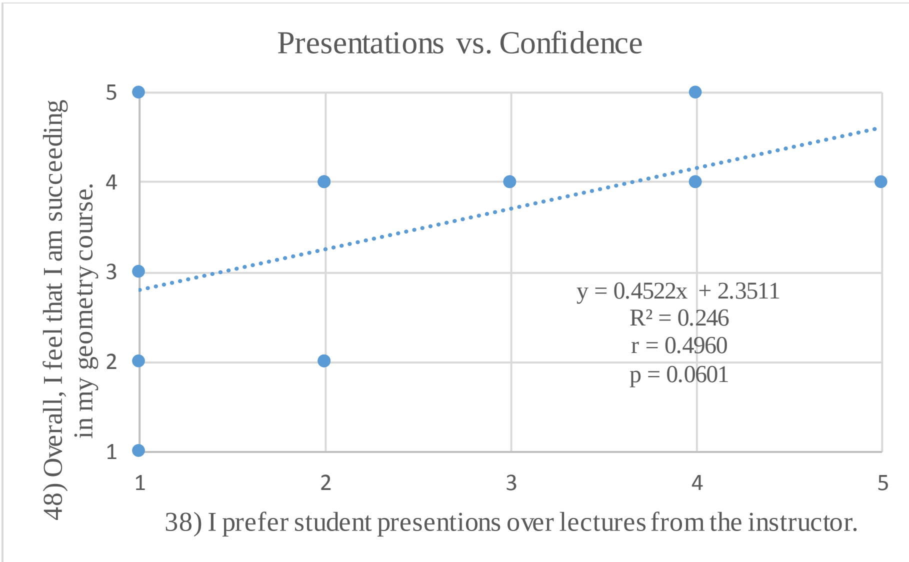
S26 ("I do not do as much homework as I should.") had the highest correlation to students' confidence. The correlation was also negative which suggests that the students who felt they were not doing enough homework were less confident within the class. A linear regression test was performed between S26 and S48. The correlation coefficient was found to be r = -0.35, which implies a weak, negative correlation. Only 13% of the variance of confidence in the course is accounted for by the students' views on the course structure (R^2 = 0.13). To further verify a significant correlation, a linear regression t-test was again performed at the 5% significance level (α = 0.05) with null and alternative hypotheses as previously stated. The test gave p = 0.19 which is greater than α = 0.05. Thus, the null hypothesis was not rejected. This implies that students' homework habits do not have a significant impact on self-confidence within the course.

Figure 4:



Presentations. The students' views on class presentations were assessed with five statements (S35 – S39). In response to S35, "I enjoy presenting in my MMM class," the mean response was 2.5. Only three out of fifteen students agreed with this statement. This suggests that students prefer lectures from the instructor which was anticipated. Students like what they are accustomed to. From the correlation matrix, the question that most strongly correlated to student's confidence was S38. In order to assess if their views towards presentations significantly affect their confidence in the class, a scatterplot was created using data from S38 and S48. The correlation coefficient was found to be r = 0.50. This suggests that presentations have a moderate, positive correlation to student confidence and 25% of the variance in students confidence in the course is accounted for by their views of presentation ($R^2 = 0.25$) . To further verify significance, a linear regression t-test was performed at $\alpha = 0.05$ with the null and alternative hypotheses as previously used. The results gave $p = 0.06$. Since $p > \alpha$, the null hypothesis failed to be rejected and it was concluded that presentations in the course have little or no impact on the students confidence.

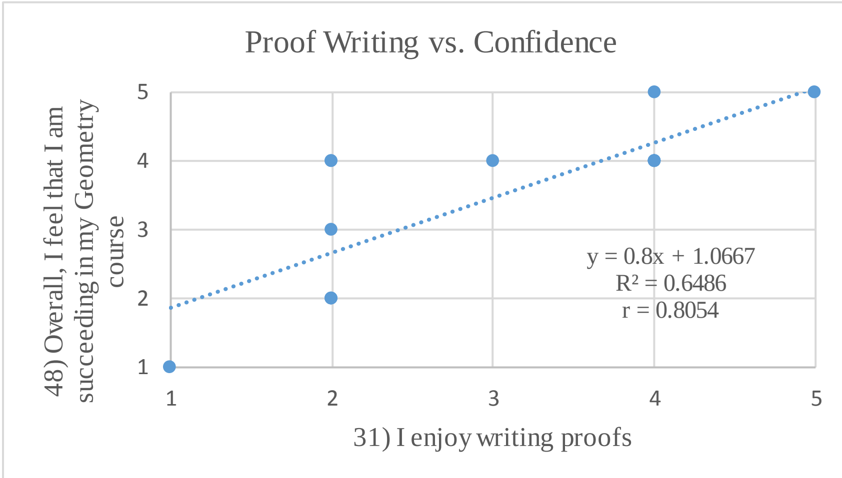
Figure 5:



Geometry Content. Students were also asked to assess their opinion of geometry in general. Statements 40 through 43 evaluate their enjoyment of the content, their view of its importance, if they find it to be interesting, and if it is difficult for them. The class mostly seemed to agree that "geometry is an important aspect of mathematics," ($\bar{x} = 4.0$) and that it is interesting ($\bar{x} = 3.5$). Likewise they disagreed to the statement "I do not enjoy the geometry content…" ($\bar{x} = 2.6$), suggesting that for the most part students enjoy geometry. From the correlation matrix, there appeared to be a weak correlation between students' views of geometry and their confidence in the course, however, these correlations were less than the required $|0.5|$. Thus, it was concluded that opinions on the course content had no significant impact on students' confidence.

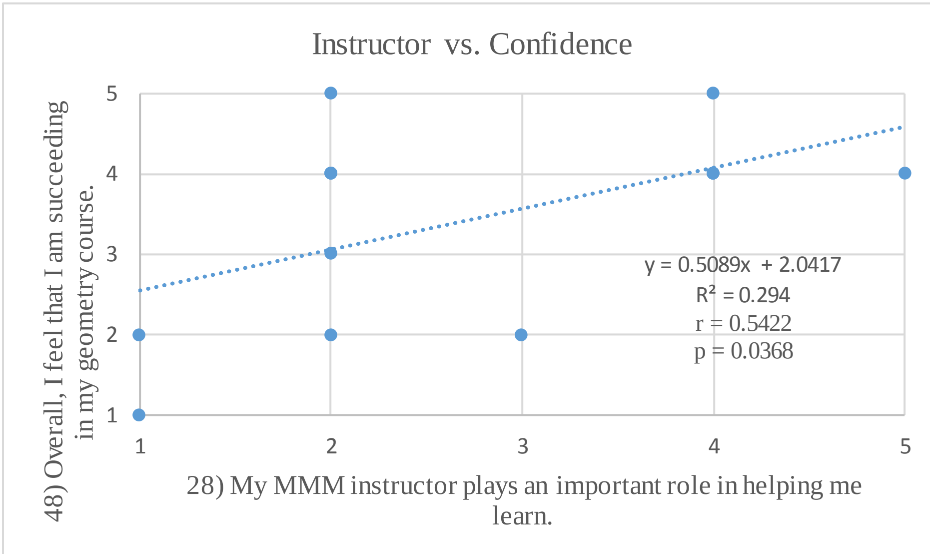
Proof Writing. To assess how the students felt about writing proofs, they were asked to respond to statements about the importance of proofs, how much they enjoy writing them, if they find proofs to be challenging, and if they agree that it is essential for mathematical understanding (S31-S34). The average response to "I enjoy writing proofs," was a neutral 3 suggesting that the class did not have a dominate opinion. Seven students agreed with the statement, seven disagreed and there was one neutral response. The class was more in agreement that proof writing is challenging ($\bar{x} = 4.2$) with only two neutral responses and 13 who agreed. For the most part, students also agreed that proofs are "essential for mathematical understanding" (S33) and important for their learning (S34) with means of 3.6 and 3.9 respectively. Of these questions, only S31 had a correlation greater than $|0.5|$. A linear regression test was performed comparing responses to S31 ("I enjoy writing proofs") and S48 ("Overall, I feel that I am succeeding in my Geometry course"). The results are given in Figure 6. Since r = 0.81 there appears to be a strong, positive correlation between the students view on proof writing and their confidence in the course. This suggests that 65% of the variance in confidence in the course is accounted for by how much the students enjoy proofs ($R^2 = 0.65$). A linear regression t-test was again used at the 5% significance level, $\alpha = 0.05$ with the same aforementioned hypotheses. The results gave $p = 2.91 \times 10^{-4}$. Since $p < \alpha$, we reject the null hypothesis. Therefore, there is sufficient evidence that there is a significant linear relationship between students' enjoyment of proof writing and their opinion of their own success in the class.

Figure 6:



Instructor. Students were asked to respond to the statements, "I have a high respect for my instructor" (S27), "My MMM instructor plays an important role in helping me learn" (S28), and "My instructor is very knowledgeable about the course content" (S30). However, the point of this study is not to evaluate the instructor, but rather see what impacts student confidence. To determine if the instructor did have an impact, the correlation matrix was analyzed and there seemed to a correlation between confidence and S28 as well as S30. Since S28 assesses students respect the instructors role within the course and it had a higher correlation to confidence than S30, it was analyzed against S48. (It was more strongly correlated to S47, however, S47 assesses confidence in proof writing which, although important, is more not what I want to focus on at this time.) A linear regression was performed with the two data sets and the resulting in a correlation coefficient r = 0.54, implying a moderate correlation. Thus, 29% the variance in students confidence in the course is accounted for by their views of presentation ($R^2 = 0.29$). Once more, a linear regression t-test was performed using the same significance level, $\alpha = 0.05$, and hypotheses. The results gave p = 0.04 which is less than α. Thus, the null hypothis was rejected. This implies that there is a significant linear relationship between students' view of the instructors role and their self-confidence in the course.

Figure 7:



Significant Results Based on Student Characteristics

Although only one class was surveyed, there are still different groups within the class. I wanted to be sure that the results I found reflected the entire class rather than just certain groups and so I looked at male students vs. female students and students who have had MMM courses before vs. those who have not. Since the data gathered from the survey was ordinal and there was a small sample size, it was necessary to use nonparametric tests instead of ANOVA or t-test. I used a Mann-Whitney U-test to determine if any of the statements had a significant difference between the two categories within each of the two sets.

The Mann-Whitney U-test depends on two assumptions:

1) Data consists of two independent random samples: X_1, X_2,…,X_n from one population and Y_1, Y_2, …,Y_n from the second population.
2) The distribution functions of the two populations are identical except for possible location parameters. (Corder & Foreman, 2014, p. 75)

Unfortunately, my data set does not precisely match the necessary assumptions. In each of the sets, the two samples are neither independent, nor random since only one class was surveyed. The small sample size also inhibits identical distributions, however, I believe that this test was the best option for my data set.

Before testing statements for significance, I found the means of each statement for the two groups to be compared (see Appendix B). I then calculated the difference in the two groups and if the difference was greater than or equal to $|1.25|$, the statement was then tested for significance at $\alpha = 0.05$.

The null hypothesis for each test was:

H_0: There is no significant difference between the two groups in response to the given statement.

The alternative hypothesis for each test was:

H_a: There is a significant difference between the two groups in response to the given statement.

Male vs. Female

The first question of the survey asks participants to give their gender. Nine students responded female, five selected male, and one student did not respond. The data collected from the student who chose not to put their gender was excluded from the following data analysis. The statements that had a significant difference based on gender are given in Figure 8 along with their corresponding means and significance from the Mann-Whiney U-test.

Figure 8:

	Statement	**Female Mean**	**Male Mean**	**Sig.**
S2	Mathematics has always been easy for me.	4.11	2.83	0.0187
S13	I feel comfortable in my MMM course.	2.56	4.00	0.0236
S14	I like how the course is structured.	2.00	3.67	0.0051
S15	The course is structure in the best way for me to learn the content.	1.89	3.50	0.0211
S16	I learn more with a MMM structure than in a lecture based course.	1.78	3.00	0.0303
S17	I prefer a lecture-based course.	4.44	2.67	0.0061
S28	My MMM instructor plays an important role in helping me learn.	2.22	3.67	0.0175
S35	I enjoy presenting in my MMM class.	1.89	3.33	0.0397
S38	I prefer student presentations over lectures from the instructor.	1.78	3.50	0.0112
S45	I prefer learning in a MMM environment.	1.56	3.50	0.0086
S47	Overall, I have a positive opinion of my MMM class.	2.11	3.50	0.0225

The results suggest that females are less comfortable in this MMM course as compared to males (S13). Females do not like the structure of the course and likewise, they do not enjoy the presentations aspect. Females are more likely to prefer a lecture-based course than their male peers (S17 and S45). The males remain a little more neutral in these two aspects. The mean response for the males on statement 17, "I prefer a lecture-based course," was 2.7 as compared to the females' 4.4. With regards to structure specifically, the male responses stayed between 3.0 and 3.7 whereas the female means were between 1.8 and 2.0 (S14-S16).

The genders also seemed to have different opinions with regards to the instructor. Males seemed to have a slightly higher opinion of the instructor as far as his role in the course (S28).

There was not a significant difference in the results for S48, suggesting there is no evidence males and females have different feelings of success within the course.

Previous MMM Experience vs. No Previous MMM Experience

Within this class, six students had previously taken a MMM course before and nine had not. The results are given in Figure 9.

Figure 9:

	Statement	Previous MMM Mean	No Previous MMM Mean	Sig.
S13	I feel comfortable in my MMM course.	4.00	2.56	0.0332
S15	The course is structure in the best way for me to learn the content.	3.50	1.89	0.0317
S18	I would enjoy taking other math classes with a similar structure.	3.50	2.20	0.0267
S22	My Geometry homework is easier than that of most of my other upper-level math courses.	3.33	2.10	0.0190
S27	I have a high respect for my instructor.	4.00	2.40	0.0332
S35	I enjoy presenting in my MMM class.	3.33	1.89	0.0393
S49	Going into the midterm I felt confident in my proof writing abilities.	4.17	2.78	0.0271
S52	When I finished the midterm, I felt confident that I did well.	4.00	2.56	0.065

These results demonstrate that students who have had a modified Moore method course before are more likely to enjoy the structure (S15, S18), respect the instructor (S27) and enjoy presenting (S35). Overall, they felt more comfortable (S13) and confident (S49, S52) in the course.

Interviews.

Out of fifteen students, five agreed to be interviewed. From those five I selected two, one female and one male. One of the two had no previous MMM experience (Student A) and the other had two other MMM courses in previous semesters (Student B). The interviews were recorded and transcribed.

As with the survey results, it was clear that the student who had never had a MMM course before greatly opposed the structure. At one point in the interview, Student A said, "I feel like I'm not really learning much through it…I feel like I would learn more and be more confident in [geometry] if we had talked more about it before." This student also reported a dislike for presentations stating, "…if I'm not really sure then I would rather not present… it is better if I feel confident in my proof." This further emphasizes the survey findings that proof writing had a significant impact on confidence. When asked what you would change about the course to increase this uncertainty, Student A responded:

> *I would definitely prefer lecture based, or at least some lecture… Cause every new packet that we get is a new topic and it's a different proof outline for each. So if maybe if at the beginning of each topic there was a general like "these are some good hints for doing these proofs" I think that would help, 'cause I always feel more confident after the first day of seeing people doing them to go on and try some on my own.*

Although this is only one students' opinion, it reflects the results found in the survey analysis that students prefer lectures. Student A provides a suggestion that could be considered in future modifications in order to increase confidence.

Student B seemed to be much happier in the course stating, "the structure of the class is pretty in line with how I learn." When asked about presentations, Student B responded:

> *I don't mind speaking to a crowd too much. I really like it when I can say something and have people talk to me and let me know if they have questions it helps me think about something in a different way possibly that I didn't think of*

> *before and try to explain it better so that I can know more about what I am proving already.*

This student found that this method of instruction was more helpful for their own personal learning. However, Student B was also more accustomed to it and had developed study habits to help prepare for each class period. This student often met with classmates to review one another's proofs and felt more comfortable asking the professor for help. In contrast, Student A stated, "I don't have time to be in his office hours all the time. That's what I have class for."

Although these students' views are quite different, Student B also suggested including more lecture time stating that it would be preferable to have, "somewhere in the middle, not all lecture, but a little bit more." When asked about changes to the class, Student B suggested changes related to one of his previous courses.

> *What that teacher did differently was he just like explained things a lot more and he gave us more hints how to do things. It was "let us do it ourselves," but it felt hands on in the way that he did it so that he was still guiding us throughout the process. He'd give us time in class to also work on it and we'd have easy access to talk to him about stuff if we had questions and I liked that a lot more than just having class proving proofs.*

Student B's suggestions show a different modification to the Moore method. Although Student B may have felt more confident in a course with this described structure, it would be interesting to see how this alternative method would impact students' confidence. Perhaps in the future, new studies will be created to investigate these seemingly minor differences.

Analysis

Of the six aspects of the MMM that were believed to impact students' confidence within this geometry course, three were shown to have little or no significant correlation: homework requirements, the geometry content, and presentations. The three remaining aspects (structure, proof writing, and the instructor) were found to be significantly correlated to students' confidence.

Structure and presentations are almost equivalent when discussing MMM courses. The course structure is almost entirely made up of student presentations. Therefore, it is interesting that presentations were not correlated to confidence in the course. This suggests that the presentation aspect of the structure was not necessarily what causes students to lose confidence. However, from the calculations on previous experience, it was shown that the students who have had previous MMM experience were found to have enjoyed both the presentations and structure. They have come to believe that it was best for their own learning. This suggests that once students overcome the discomfort of this method and become used to inquiry-based learning, they will appreciate the design and its impact on their own understanding.

Although not initially anticipated, the instructor's role in the course was found to have a significant correlation to students' confidence. However, this also seems to relate back to students' views on the course structure. Students who did not enjoy the structure would likely also not like that the instructor is playing less of a role in a MMM class. If they prefer lecture style courses, that implies that they believe the instructor should be in front of the class rather than observing from the back.

Proof writing, compared to the other five aspects, had the strongest correlation. The more students enjoyed proofs, the more they felt confident in the course. Since students' have to present these proofs in front of the entire class, it can be intimidating. A student unsure of a proof, is not likely to be confident presenting to his peers and instructor. However, results from the Mann-Whitney U-tests showed that students who have more experience with MMM were more confident in their proof writing abilities. Although it cannot be verified with the data collected, this previous MMM experience may also imply more experience in proof-intensive courses. In either case, as students have more practice with proofs and more experience in MMM courses, their confidence has been shown to increase within their MMM classes.

Results also suggest that males are more satisfied in a MMM course than females. Of the responses to the statements that significantly differed between males and females, the males remained more neutral in their views on structure. The females seemed to have stronger opinions against the course entirely. This difference in opinion among gender cannot be determined from this study. Further exploration would be needed in future investigations. However, it should be noted that there was not a significant difference among the genders in regards to confidence within the course. The females may have been less comfortable (S13), but their self-confidence is comparable to their male peers.

Limitations and Conclusion

There are several limitations to this study that must be acknowledged. The first is that I was only able to survey and evaluate one class with fifteen students taught by a single professor. Because of this, I had very little statistical data and the quantitative data I did gather was very specific to this particular class. Further, although I can discuss students' confidence in this MMM course, I cannot make any assertions regarding different modifications and techniques.

There are a few aspects within my study that a more experienced quantitative researcher may have done differently. I used 23 different hypothesis tests in this study and as a result there is a high probability for Type I error. Further, I believe that similar, if not the same, results could have been found with a much smaller survey that more precisely assessed students' views and confidence within the course. In doing so I would be able to perform a decreased number of hypothesis tests and thus increase the validity of my findings.

With the limited resources available to me, I could not create a flawless study. However, I hope to promote the creation of future studies in this field. I would recommend an analysis of two MMM courses that employ different modifications. If the same course was taught by the same instructor using two different techniques, you could minimize the effect of confounding variables.

Although the results may be overshadowed by limitations, I believe that I have shown that students' confidence is impacted by their perception of the course structure, their views towards proofs, the instructor, and especially their experience. In order to promote student confidence, teachers need to know a little about the educational history of the class. New teaching strategies should be introduced like new content: building off of previous knowledge. From the interviews, I could see that even students who enjoyed MMM preferred to have more lectures included within the course. In general, this is how students have been taught to learn from an early age. To promote both academic achievement and student confidence, we need to be mindful of their needs.

Recommendations for Teachers

It is important for teachers, especially of mathematics, to keep students confident in themselves and their abilities. Although this study only focuses on an upper-level collegiate mathematics course, I believe that the principles can be applied to many levels of education.

Students are more confident when they are comfortable. They are more comfortable when they are in environment that they are familiar with. However, inquiry-based learning has been shown to increase students' understanding and academic success (Cooper et al., 2012). It forces students to think for themselves and when they come to new conclusions on their own, they are more likely to retain that information. This student-centered technique can be shocking to students, but it does not have to be. If students start seeing inquiry-based instruction at an early age, they are more likely to enjoy it as they progress through their education. I would encourage teachers who may be new to IBL to try gradually implementing these strategies into their classroom while carefully monitoring students' progress and confidence.

Closing Remarks

This study has had a greater impact on me than I ever thought it could. I have discovered that I enjoy looking at research, proposing ideas, and analyzing results. Mathematically, I have learned new methods of analyzing data and when they are appropriate, as well as the limits of different data sets and tests. Most importantly though, I have learned the benefits of inquiry based instruction and confidence among students. I am excited to have a chance to apply my research to my future classroom and I am now looking forward to future projects. Students need knowledgeable and professional teachers. I am hoping that this study and continued research will better me as an educator so that I can help my students in every possible way.

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Appendix A

Modified Moore Method Survey

Personal Information
Please circle the appropriate response

Gender: M F

Age: 17 or younger 18 19 20 21 22 23 24 or older

Class: Freshman Sophomore Junior Senior

Please fill in each blank to the best of your knowledge

Major: ____

Minor (if applicable): ____________________________________

Total number of mathematics courses completed in college past Calculus 2: ____

Please circle your average letter grade in your college math courses:

A+ A A- B+ B B- C+ C C- D F

Previous MMM Experience
For the purposes of this study, we will define **Modified Moore Method (MMM)** to be a form of mathematical instruction in which the professor provides problems for the students to answer or prove and then present their solutions to the class. These presentations must take at least 50% of the class time in order for the course to be considered MMM.

Is Geometry the first course you have taken in college with a Modified Moore Method (MMM), structure?

Yes No

If no, how many other courses have you had in a MMM format?

0 1 2 3 4 5 or more

Please list the MMM courses:

____

____

____

Survey Questions
Please indicate to what degree you agree or disagree with the following statements:

		SD		**N**		**SA**
1)	In general, I enjoy mathematics.	1	2	3	4	5
2)	Mathematics has always been easy for me.	1	2	3	4	5
3)	My friends have always been better at math than me.	1	2	3	4	5
4)	Mathematics is challenging.	1	2	3	4	5
5)	There is only one way to correctly answer a math problem.*	1	2	3	4	5
6)	Everything important in mathematics has already been discovered.*	1	2	3	4	5
7)	Memorization is key to succeeding in mathematics.*	1	2	3	4	5
8)	There is an aspect of creativity in mathematics.*	1	2	3	4	5
9)	Conceptual understanding is important for success in math.	1	2	3	4	5
10)	I am good at math.	1	2	3	4	5
11)	I feel confident in my mathematical abilities.	1	2	3	4	5
12)	I am successful in mathematics.	1	2	3	4	5
13)	I feel comfortable in my MMM course.	1	2	3	4	5
14)	I like how the course is structured.	1	2	3	4	5
15)	The course is structure in the best way for me to learn the content.	1	2	3	4	5
16)	I learn more with a MMM structure than in a lecture based course.	1	2	3	4	5
17)	I prefer a lecture-based course.	1	2	3	4	5
18)	I would enjoy taking other math classes with a similar structure.	1	2	3	4	5
19)	My MMM course challenges me more than my lecture-based mathematics courses.	1	2	3	4	5
20)	I have to put more time into my MMM course in order to learn.	1	2	3	4	5
21)	Compared to my lecture-based classes, I spend more time outside of class working on my MMM homework.	1	2	3	4	5
22)	My Geometry homework is easier than that of most of my other upper-level math courses.	1	2	3	4	5

		SD		N		SA
23)	I would rather do my Geometry homework than the homework from my other upper-level math courses.	1	2	3	4	5
24)	It is sometimes difficult to motivate myself to do my Geometry homework.	1	2	3	4	5
25)	I do not do my Geometry homework unless I plan on presenting during the next class period.	1	2	3	4	5
26)	I do not do as much homework as I should for this class.	1	2	3	4	5
27)	I have a high respect for my instructor.	1	2	3	4	5
28)	My MMM instructor plays an important role in helping me learn.	1	2	3	4	5
29)	It is primarily my own responsibility to learn in this course.	1	2	3	4	5
30)	My instructor is very knowledgeable about the course content.	1	2	3	4	5
31)	I enjoy writing proofs.	1	2	3	4	5
32)	I find proof writing to be challenging.	1	2	3	4	5
33)	Proof writing is essential for mathematical understanding.	1	2	3	4	5
34)	It is important for me to learn how to write proofs.	1	2	3	4	5
35)	I enjoy presenting in my MMM class.	1	2	3	4	5
36)	Presentations are not necessary for me to learn the content.	1	2	3	4	5
37)	Presentations are easy.	1	2	3	4	5
38)	I prefer student presentations over lectures from the instructor.	1	2	3	4	5
39)	My classmates are encouraging during the daily presentations in my MMM class.	1	2	3	4	5
40)	I do not enjoy the geometry content of the course.	1	2	3	4	5
41)	Geometry is an important aspect of mathematics.	1	2	3	4	5
42)	I find geometry to be interesting.	1	2	3	4	5
43)	Geometry is difficult for me.	1	2	3	4	5
44)	My MMM course encourages self-discovery.	1	2	3	4	5
45)	I prefer learning in a MMM environment.	1	2	3	4	5

		SD		**N**		**SA**
46)	Compared to my lecture based math courses, I feel like my MMM class covers more content.	1	2	3	4	5
47)	Overall, I have a positive opinion of my MMM class.	1	2	3	4	5
48)	Overall, I feel that I am succeeding in my Geometry course.	1	2	3	4	5
49)	Going into the midterm I felt confident in my proof writing abilities.	1	2	3	4	5
50)	The course content prepared me for the midterm.	1	2	3	4	5
51)	The midterm was reflective of the work we did in class.	1	2	3	4	5
52)	When I finished the midterm, I felt confident that I did well.	1	2	3	4	5

*Adapted from Schoenfeld [1989].

Time Spent on Course Work

Please circle an appropriate response for each question below.

On average, how many hours outside of class do you spend working on your MMM course each week?

0 hours 0.5 hours 1 hour 1.5 hours 2 hours

2.5 hours 3 hours 4 hours 5 hours 6+ hours

On average, how many hours outside of class do you spend working on a single 3000+ level math courses each week?

0 hours 0.5 hours 1 hour 1.5 hours 2 hours

2.5 hours 3 hours 4 hours 5 hours 6+ hours

Would you be willing to participate in an interview to discuss your MMM course?

Yes No

If yes, please include your contact information below.

Name: ____

Email: ____

Thank you so much for your participation!!

Appendix B

Statement Number	Class Mean	Standard Deviation	Female Mean	Male Mean	Previous MMM Mean	No Previous MMM Mean
S1	4.47	0.50	4.67	4.00	4.50	4.44
S2	3.60	1.02	*4.11	*2.60	3.83	3.44
S3	2.40	0.71	2.22	2.60	2.50	2.33
S4	3.80	0.75	3.67	4.20	3.67	3.89
S5	1.67	0.87	1.78	1.60	1.33	1.89
S6	1.93	0.68	1.89	2.20	1.67	2.11
S7	1.93	0.68	1.67	2.40	1.67	2.11
S8	4.13	0.50	4.11	4.20	4.33	4.00
S9	4.33	0.79	4.44	4.20	4.00	4.56
S10	4.00	0.73	4.33	3.40	4.00	4.00
S11	3.93	0.77	4.22	3.40	4.17	3.78
S12	4.00	0.76	4.38	3.40	4.20	3.89
S13	3.13	1.20	*2.56	*4.20	**4.00	**2.56
S14	2.67	1.19	*2.00	*4.00	3.33	2.22
S15	2.53	1.26	*1.89	*3.60	**3.50	**1.89
S16	2.27	1.24	*1.78	*3.40	3.00	1.78
S17	3.73	1.18	*4.44	*2.40	3.50	3.89
S18	2.73	1.06	2.22	3.60	**3.50	**2.22
S19	3.33	0.94	3.33	3.40	3.83	3.00
S20	3.93	0.68	3.89	4.20	4.00	3.89
S21	3.40	1.02	3.11	4.20	3.50	3.33
S22	2.60	0.95	2.22	2.80	**3.33	**2.11
S23	2.47	1.09	2.00	3.00	2.50	2.44
S24	4.20	0.54	4.33	4.00	4.00	4.33
S25	3.80	1.11	4.00	3.40	3.83	3.78
S26	3.80	0.75	4.00	3.40	3.33	4.11
S27	3.07	1.24	2.89	3.60	**4.00	**2.44
S28	2.80	1.22	*2.22	*4.00	3.50	2.33
S29	4.47	0.62	4.44	4.40	4.50	4.44
S30	3.53	0.72	3.22	4.20	3.83	3.33
S31	3.00	1.15	2.67	3.40	3.33	2.78
S32	4.20	0.65	4.11	4.40	4.00	4.33
S33	3.60	1.02	3.67	3.60	4.00	3.33
S34	3.93	1.00	4.00	3.80	4.33	3.67
S35	2.47	1.20	*1.89	*3.40	**3.33	**1.89
S36	3.07	1.06	3.33	2.40	3.00	3.11
S37	2.87	0.88	2.56	3.40	3.17	2.67
S38	2.47	1.26	*1.78	*3.80	3.17	2.00
S39	4.00	0.97	4.00	4.00	4.50	3.67
S40	2.60	0.80	2.67	2.60	2.33	2.78
S41	4.00	0.82	4.11	3.80	3.67	4.22
S42	3.53	0.96	3.33	3.80	3.67	3.44
S43	3.13	1.26	3.00	3.80	2.33	3.67
S44	3.40	1.08	2.89	4.20	4.00	3.00
S45	2.33	1.35	*1.56	*3.80	3.17	1.78

Statement Number	Class Mean	Standard Deviation	Female Mean	Male Mean	Previous MMM Mean	No Previous MMM Mean
S46	2.27	0.93	1.89	3.00	2.50	2.11
S47	2.67	1.40	*2.11	*4.00	3.50	2.11
S48	3.47	1.15	3.22	3.80	4.17	3.00
S49	3.33	1.14	3.00	3.80	**4.17	**2.78
S50	3.40	1.02	3.11	4.00	4.00	3.00
S51	3.87	0.72	3.78	4.20	4.33	3.56
S52	3.13	0.96	2.67	3.80	**4.00	**2.56

*The means of the questions in which the male and female groups had a significant difference of opinion according to the Mann-Whitney U test.

** The means of the questions in which the group with previous MMM experience was significantly different from that of the group without MMM experience according to the Mann-Whitney U test.