Abstract

Consider K to be an arbitrary field, and $P(n_1, \ldots, n_m)$ be the ideal of polynomials given by

P$(n_1, \ldots, n_m) = \{f(x_1, \ldots, x_m) : f(x_1, \ldots, x_m) \in K[x_1, \ldots, x_m], f(t^{n_1}, , t^{n_m}) = 0$, where t is transcendental over K$\}$.

In 1970, J. Herzog showed that the least upper bound on the number of generators of K, for $m = 3$, is 3. It can be lowered to two, if n_1, n_2, n_3 satisfy a few symmetry conditions. Following that, Bresinsky in 1975, showed that the lowest upper bound on the number of generators of $P(n_1, \ldots, n_m)$, can be arbitrarily large if $m \geq 4$.Recent work by Herzog and Stamate provides a closed form for the number of generators for the semigroup in Bresinskys example showing that this number is arbitrarily large but even (precisely $2n$, where n is built into Bresinsky's semigroup and can be any natural number).

Since then a lot of progress has been made in investigating, and finding a closed form for the number of generators of theideal of relations for m greater than or equal to 4. All established work in the field produced examples where this number is always an even number.
However, in 2017, Stamate considers a semigroup suggested by Backelin, which has the following structure.

$$H = \langle r(3n+2),\ r(3n+2)+3,\ r(3n+2)+3n+1,\ r(3n+2)+3n+2\rangle$$

where $n \geq 2$, and $r \geq 3n+2$.

Stamatereports that computations using Singular and GAP indicate that the number of generators for this semigroup is 3n+4, which can be an odd number.

The purpose of this project is to theoretically verify that result. In doing so, the project not only answers a fundamental question in semigroup theory, but also fills the vacuum caused by the lack of any examples with an odd number of generators, thereby completing a 43-year-old question.