Numerical evidence towards establishing a new relation satisfied by the coefficients of the Hilbert quasi-polynomial of a polynomial ring with general grading

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Our research is centered around studying the Hilbert quasi-polynomial of a polynomial ring R in finitely many variables over a field K, with general grading. In our case, the Hilbert quasi-polynomial of R describes the Hilbert function of R. The Hilbert function is the function $f(n)$ such that $f(n)$ equals the K-dimension of the vector space of all polynomials from R of degree n. The Hilbert theory says that $f(n)$ is a quasi-polynomial in n. In particular, let d_i=degree of the variable x_i in R, $i = 1, \dots, m$, and let D=lcm$(d_1, \dots, d_m)$. We are interested in the polynomial $h(n)$ such that $h(nD) = f(nD)$, for all n. Our work is around gathering numerical evidence for the claim that $h(n-1)$ is a polynomial with nonnegative coefficients. We are running tests and examples through algebraic software such as Singular and Polymake to gather examples in this direction. Gathering such evidence would give an indication that this feature of the $h(n)$ is true and provide ideas on how to approach a proof of this claim. This claim is important in the theory of Frobenius complexity. In addition, it would provide a surprising set of new features of the Hilbert quasi-polynomial, of interest in enumerative combinatorics and discrete geometry.